

## Handout for 2020-02-24

**Problem 1.** Find the distance  $d$  between the point  $P(1, -2, 2)$  and the line  $\mathbf{r}(t) = \langle 3 + 3t, 2 - t, 5t \rangle$ .

(a) Solution 1:

$$\begin{aligned} D^2 &= (2 + 3t)^2 + (4 - t)^2 + (5t - 2)^2 \\ &= 35t^2 - 16t + 24 \\ \frac{d}{dt}(D^2) &= 70t - 16 = 0 \\ t &= 8/35 \\ d = D_{\min} &= \sqrt{35(8/35)^2 - 16(8/35) + 24} = 2\sqrt{194/35}. \end{aligned}$$

(b) Solution 2:

$$\begin{aligned} 3(x - 1) - (y + 2) + 5(z - 2) &= 0 \\ 3x - y + 5z - 15 &= 0 \\ 3(3 + 3t) - (2 - t) + 5(5t) - 15 &= 0 \\ t &= 8/35 \\ d &= \sqrt{(3 + 3(8/35) - 1)^2 + (2 - (8/35) + 2)^2 + (5(8/35) - 2)^2} = 2\sqrt{194/35}. \end{aligned}$$

(c) Solution 3:

$$\begin{aligned} \langle 1, -2, 2 \rangle - \langle 3, 2, 0 \rangle &= \langle -2, -4, 2 \rangle \\ \langle -2, -4, 2 \rangle \times \langle 3, -1, 5 \rangle &= \langle -18, 16, 14 \rangle \\ |\langle -18, 16, 14 \rangle| &= 2\sqrt{194} \\ |\langle 3, -1, 5 \rangle| &= \sqrt{35} \\ d &= 2\sqrt{194/35}. \end{aligned}$$

(d) Solution 4:

$$\begin{aligned} \langle 1, -2, 2 \rangle - \langle 3, 2, 0 \rangle &= \langle -2, -4, 2 \rangle \\ (\langle 3, -1, 5 \rangle \times \langle -2, -4, 2 \rangle) \times \langle 3, -1, 5 \rangle &= \langle 94, 132, -30 \rangle = 2\langle 47, 66, -15 \rangle \\ \frac{\langle 47, 66, -15 \rangle \cdot \langle -2, -4, 2 \rangle}{|\langle 47, 66, -15 \rangle|} &= -388/\sqrt{6790} \\ d &= |-388/\sqrt{6790}| = 2\sqrt{194/35}. \end{aligned}$$

(e) Solution 5:

$$\begin{aligned} \langle 2 + 3t, 4 - t, 5t - 2 \rangle \cdot \langle 3, -1, 5 \rangle &= 0 \\ 35t - 8 &= 0 \\ t &= 8/35 \\ d &= \sqrt{(3 + 3(8/35) - 1)^2 + (2 - (8/35) + 2)^2 + (5(8/35) - 2)^2} = 2\sqrt{194/35} \end{aligned}$$

(f) Solution 6:

$$\begin{aligned}\langle 1, -2, 2 \rangle - \langle 3, 2, 0 \rangle &= \langle -2, -4, 2 \rangle \\ \frac{\langle 3, -1, 5 \rangle \cdot \langle -2, -4, 2 \rangle}{\langle 3, -1, 5 \rangle \cdot \langle 3, -1, 5 \rangle} \langle 3, -1, 5 \rangle &= \frac{8}{35} \langle 3, -1, 5 \rangle \\ \langle -2, -4, 2 \rangle - \frac{8}{35} \langle 3, -1, 5 \rangle &= \left\langle -\frac{94}{35}, -\frac{132}{35}, \frac{6}{7} \right\rangle \\ \left| \left\langle -\frac{94}{35}, -\frac{132}{35}, \frac{6}{7} \right\rangle \right| &= 2\sqrt{194/35}.\end{aligned}$$